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Reverse time migration image improvement using integral transforms

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Content

Integral transforms

An integral transform is a map from one function $f(\mathbf{x})$, to another, F($\mathbf{k}$) (Deakin,1985). The integral transform of a function $f(\mathbf{x})$ is denoted by $\mathscr{T}\{f(\mathbf{x})\} = F(\mathbf{k})$, and defined by

$$F(\mathbf{k}) = \int_S K(\mathbf{x}, \mathbf{k}) f(\mathbf{x}) d\mathbf{k} \tag{1}$$

where
$\mathbf{x} = (x_1, x_2, ..., x_n)$
$\mathbf{k} = (k_1, k_2, ..., k_n)$
$K(\mathbf{x}, \mathbf{k})$= Kernel of the transform
$S \subset \mathbb{R}^n$

Integral transforms

Some integral transforms

Fourier transform $\mathscr{F}\{f(\mathbf{x})\} = F(\mathbf{k}) = \frac{1}{(2\pi)^{n/2}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(\mathbf{x}) e^{-i(\mathbf{k}\cdot\mathbf{x})} d\mathbf{x}$

Mellin transform $\mathscr{M}_{-}\{f(\mathbf{r})\} = \tilde{F}_{-}(p) = \int_{a}^{\infty} \left(r^{p-1} - \frac{a^{2p}}{r^{p+1}} \right) f(r) dr$

Hankel transfom $\mathscr{H}_n = \int_{0}^{\infty} r J_n(kr) f(r) dr$

Radon transform $\hat{f}(p, \mathbf{u}) = \mathscr{R}\{f(\mathbf{x})\} = \int_{-\infty}^{\infty} f(\mathbf{x}) \delta(p - \mathbf{x} \cdot \mathbf{u}) d\mathbf{x}$

Integral transforms

All integral transforms presented above are linear integral transforms, i.e, $\forall \alpha, \beta \in \mathbb{R}$, $\forall \mathbf{x} \in \mathbb{R}^n$ and f, g functions on $\mathbb{R}^n$:

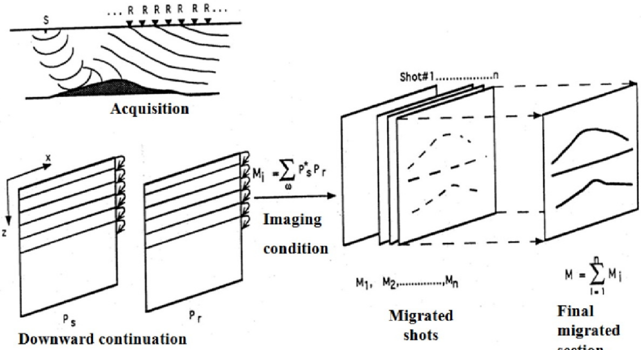
$$\begin{aligned} \mathscr{T}\{\alpha f(\mathbf{x}) + \beta g(\mathbf{x})\} &= \int_S K(\mathbf{x}, \mathbf{k})(\alpha f(\mathbf{x}) + \beta g(\mathbf{x})) d\mathbf{k} \\ &= \alpha \mathscr{T}\{f(\mathbf{x})\} + \beta \mathscr{T}\{g(\mathbf{x})\} \end{aligned}$$

Some areas of application of integral transfoms

- Fluid mechanics.
- Signal and image processing.
- Quantum mechanics.
- Geophysics.
    - One way wave equation migration (OWWE).
        - Phase shift migration.
        - PSPI and SS migration.
    - Kirchhoff migration.

Problem statement

Reverse time migration (RTM)



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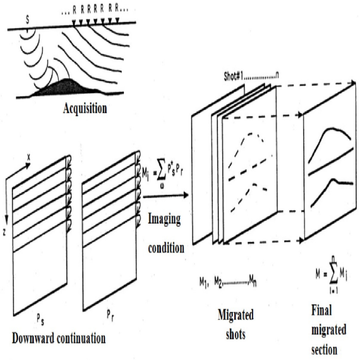
Problem statement

Reverse time migration (RTM)



Acoustic wave equation

$$\frac{1}{c(\mathbf{x},\mathbf{z})^2}\frac{\partial^2 u(\mathbf{x},\mathbf{z},t)}{\partial t^2} - \nabla^2 u(\mathbf{x},\mathbf{z},t) = s(\mathbf{x},\mathbf{z},t)$$

1. Forward propagation of the source wavefield.
2. Backward propagation of the receivers wavefield.
3. Apply a criterion to construct the seismic image (Imaging condition).

Problem statement

Cross correlation imaging condition

$$I_{CC}(x, z) = \sum_{j=1}^{s_{max}} \sum_{i=1}^{t_{max}} S(x, z; t_i; s_j) R(x, z; t_i; s_j)$$

where

S: Source wavefield

R: Receiver wavefield

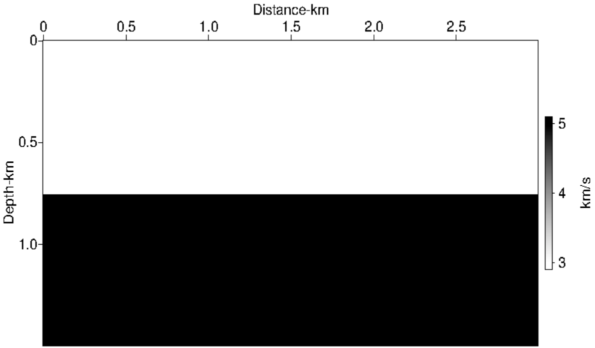
z: Depth

x: Distance

t: Time

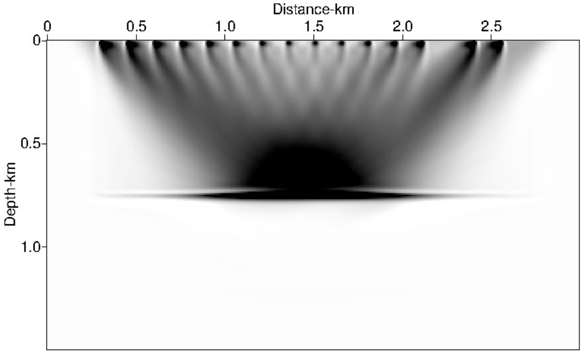
Problem statement

Velocity model of two layers synthetic model



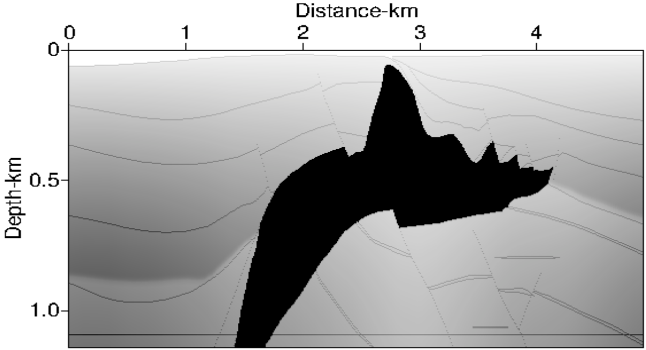
Problem statement

Cross correlation image of two layers model



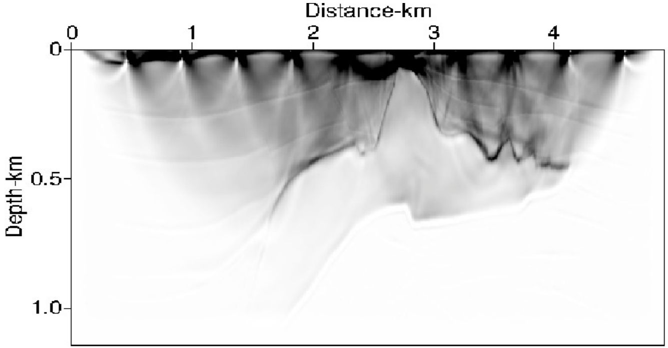
Problem statement

Velocity model $2D$ SEG/EAGE salt model



Problem statement

Cross correlation image of $2D$ SEG/EAGE salt model



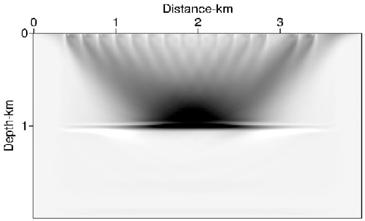
Methods to eliminate the artifacts

1. Wavefield propagation approches (Loewenthal, 1983, 1987, Baysal, 1984, Fletcher, 2005).
2. Imaging condition approches (Valenciano and Biondi, 2003 Kaelin et al, 2006, Guitton, 2007, Liu, 2011, Whitmore, 2012, Pestana et al, 2013, Shragge, 2014).
3. Post-imaging condition approches (Youn, 2001, Guitton et al, 2006).

Imaging condition approches

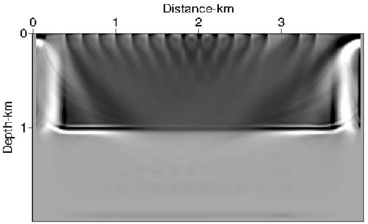
Source illumination imaging condition

$$I_{sill}(x,z) = \frac{\sum_t S(x,z,t)R(x,z,t)}{\sum_t S^2(x,z,t)}$$



Receiver illumination imaging condition

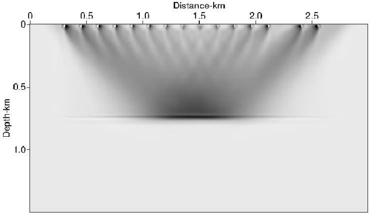
$$I_{rill}(x,z) = \frac{\sum_t S(x,z,t)R(x,z,t)}{\sum_t R^2(x,z,t)}$$



Imaging condition approches

Inverse scattering imaging condition

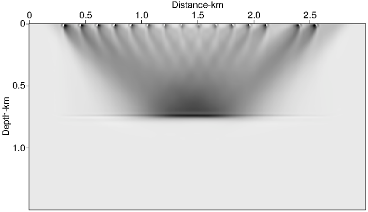
$$I_{IS}(x,z) = \sum_{t=0}^{T_{max}} \left[\nabla S(x,z,t) \cdot \nabla R(x,z,t) - \frac{1}{c(x,z)^2} \frac{\partial S(x,z,t)}{\partial t} \frac{\partial R(x,z,t)}{\partial t} \right]$$



Imaging condition approches

Impedance sensitivity kernel imaging condition

$$I_k(x,z) = \frac{1}{v^2(x,z)} \int \frac{\partial}{\partial t} P_F(x,z,t) \frac{\partial}{\partial t} P_B(x,z,t) dt + \int \nabla P_F(x,z,t) \cdot \nabla P_B(x,z,t) dt$$

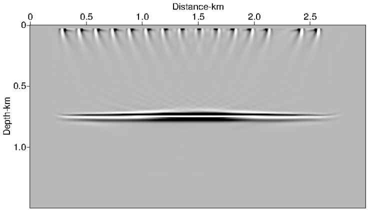


Post-imaging condition approches

Laplacian filtering

$$I_{LP}(x,z) = \frac{\partial}{\partial x^2} I_{cc}(x,z) + \frac{\partial}{\partial z^2} I_{cc}(x,z)$$

$I_{cc}(x,z)$: Cross correlation image.



Laguerre-Gauss transform

The Laguerre-Gauss transform of $I(x, y)$ is given by (Wang et al, 2006, Guo et al, 2006):

$$\tilde{I}(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} LG(f_x, f_y) I(f_x, f_y) e^{2\pi i (f_x x, f_y y)} df_x df_y \tag{2}$$

where

$$LG(f_x, f_y) = (f_x + i f_y) e^{-(f_x^2 + f_y^2)/\omega^2} = \rho e^{-(\rho^2/\omega^2)} e^{i\beta} \tag{3}$$

$\rho = \sqrt{f_x^2 + f_y^2}$, $\beta = \tan^{-1}\left(\frac{f_y}{f_x}\right)$ are the polar coordinates in the spatial frequency domain.

Laguerre-Gauss transform

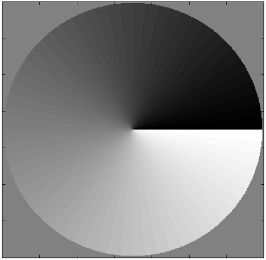
$$\tilde{I}(x,y) = |\tilde{I}(x,y)|e^{i\theta(x,y)} = I(x,y) * LG(x,y)$$

From equation (3) we obtain

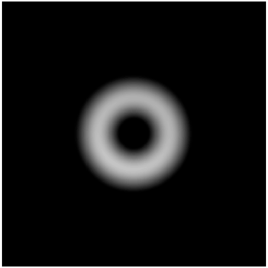
$$\begin{aligned} LG(x,y) &= \mathscr{F}^{-1}\{LG(f_x,f_y)\} = (i\pi^2\omega^4)(x+iy)e^{-\pi^2\omega^2(x^2+y^2)} \\ &= (i\pi^2\omega^4)[re^{-\pi^2 r^2\omega^2}e^{i\alpha}] \end{aligned}$$

where $r = \sqrt{x^2+y^2}$, $\alpha = \tan^{-1}\left(\frac{y}{x}\right)$ are the spatial polar coordinates.

Laguerre-Gauss transform



Spiral phase function

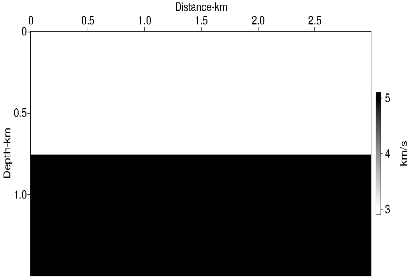


Toroidal amplitud

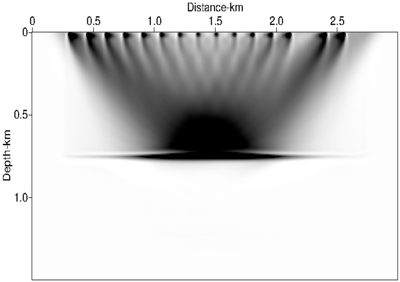
Figure: Laguerre Gauss Filter (Wang et al, 2006)

Two layers synthetic model

Velocity model



Cross correlation image



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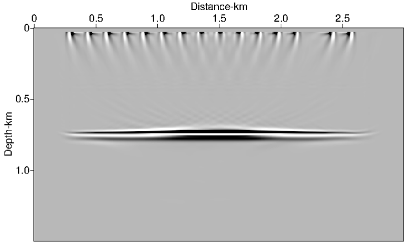
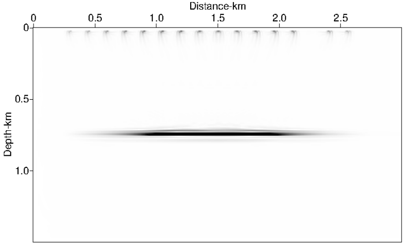
Two layers synthetic model

Laplacian image

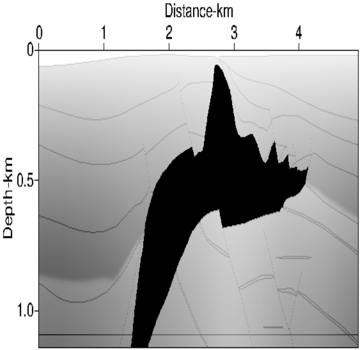


Laguerre Gauss image

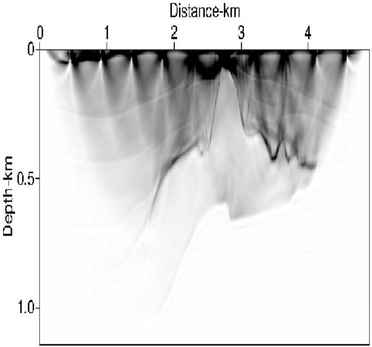


$2D$ SEG/EAGE salt model

Velocity model

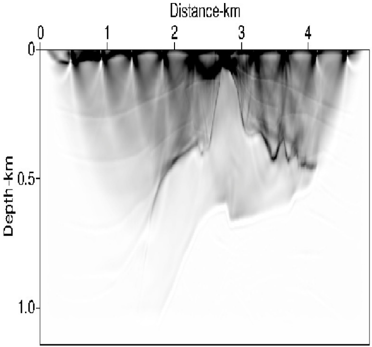


Cross correlation image

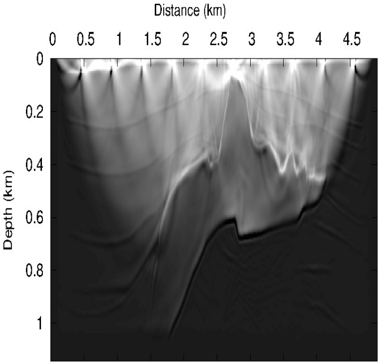


2*D* SEG/EAGE salt model

Cross correlation image

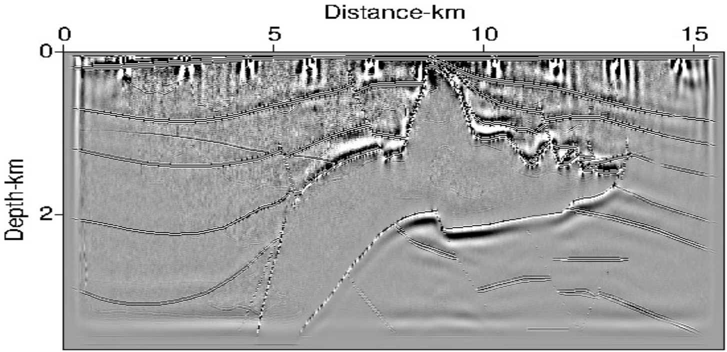


Scaled Cross correlation image



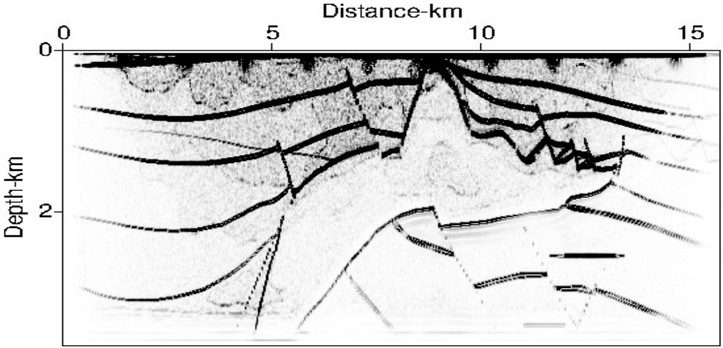
2D SEG/EAGE salt model

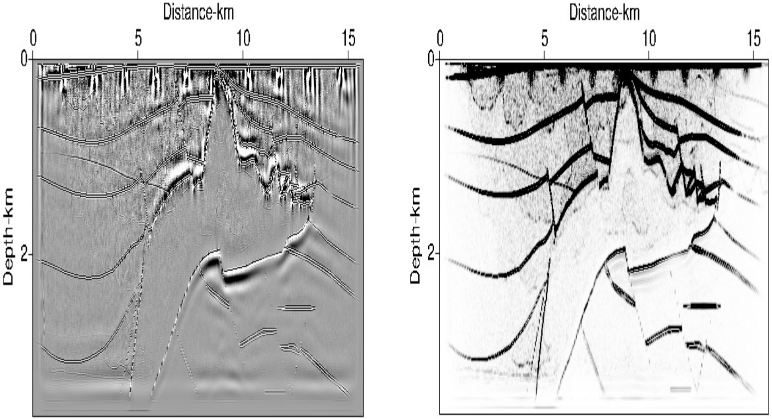
Laplacian image



$2D$ SEG/EAGE salt model

Laguerre-Gauss image





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Future work

- Write a paper reporting the results obtained with this post-imaging condition.
- Report the advances obtained in the imaging condition to ICP.
- Use computer cluster to compute the RTM algorithms parallelized.
- Measure the accuracy of the image obtained by Laguerre-Gauss Filtering compared with the true image (velocity model).
- Implement Laguerre-Gauss transform to modify or propose a new imaging condition.

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